United Kingdom
Mathematics Trust

Senior Mathematical Challenge

10 – 11 November 2021

Organised by the United Kingdom Mathematics Trust

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Candidates must be full-time students at secondary school or FE college.
England & Wales: Year 13 or below
Scotland: S6 or below
Northern Ireland: Year 14 or below

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **90 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark A, B, C, D, E on the Answer Sheet for each question. Mark only one option, boldly, within the box.
5. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all markings, including bits of eraser stuck to the page, and interpret the mark in its own way.
6. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
7. **Scoring rules:** All candidates start with 25 marks; 0 marks are awarded for each question left unanswered; 4 marks are awarded for each correct answer; 1 mark is deducted for each incorrect answer (to discourage guessing).
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.
9. To accommodate candidates sitting at other times, please do not discuss the paper on the internet until 08:00 BST on Friday 12 November.

Enquiries about the Senior Mathematical Challenge should be sent to:

UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS2 9JT

☎ 0113 365 1121 challenges@ukmt.org.uk www.ukmt.org.uk

1. Cicely had her 21st birthday in 1939.

When did she have her 100th birthday?

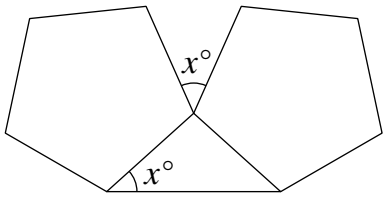
A 2020 B 2019 C 2018 D 2010 E 2008

2. The sequence, formed from the sequence of primes by rounding each to the nearest ten, begins 0, 0, 10, 10, 10, 10, 20, 20, 20, 30, ...

When continued, how many terms in this sequence are equal to 40?

A 1 B 2 C 3 D 4 E 5

3. The diagram shows two congruent regular pentagons and a triangle. The angles marked $x°$ are equal.



What is the value of x?

A 24 B 30 C 36 D 40 E 45

4. The positive integer k is a solution of the equation $(k \div 12) \div (15 \div k) = 20$.

What is the sum of the digits of k?

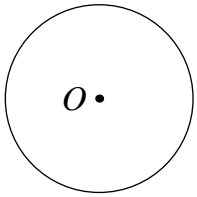
A 15 B 12 C 9 D 6 E 3

5. The sum of four consecutive primes is itself prime.

What is the largest of the four primes?

A 37 B 29 C 19 D 13 E 7

6. Three points, P, Q and R are placed on the circumference of a circle with centre O. The arc lengths PQ, QR and RP are in the ratio 1 : 2 : 3.



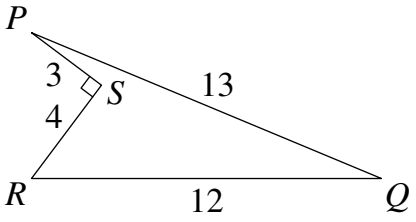
In what ratio are the areas of the sectors POQ, QOR and ROP?

A 1 : 1 : 1 B 1 : 2 : 3 C $1 : \pi : \pi^2$ D 1 : 4 : 9
E 1 : 8 : 27

7. Which of these numbers is the largest?

A 2^{5000} B 3^{4000} C 4^{3000} D 5^{2000} E 6^{1000}

8. What is the area of the region inside the quadrilateral $PQRS$?



A 18 B 24 C 36 D 48
E more information needed

9. Alison has a set of ten fridge magnets showing the integers from 0 to 9 inclusive.

In how many different ways can she split the set into five pairs so that the sum of each pair is a multiple of 5?

A 1 B 2 C 3 D 4 E 5

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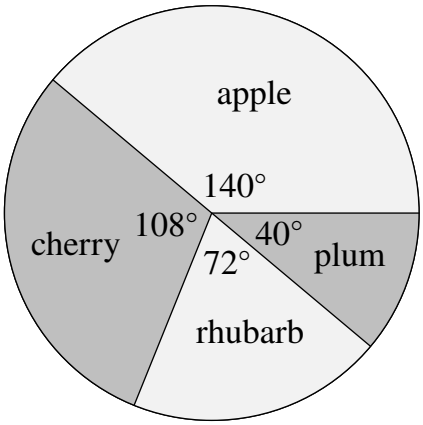
10. In a survey, people were asked to name their favourite fruit pie. The pie chart shows the outcome. The angles shown are exact with no rounding.



What is the smallest number of people who could have been surveyed?

A 45 B 60 C 80 D 90 E 180

11. Alitta claims that if p is an odd prime then $p^2 - 2$ is also an odd prime.

Which of the following values of p is a counterexample to this claim?

A 3 B 5 C 7 D 9 E 11

12. For how many positive integers N is the remainder 6 when 111 is divided by N?

A 5 B 4 C 3 D 2 E 1

13. Which of these is the mean of the other four?

A $\sqrt{2}$ B $\sqrt{18}$ C $\sqrt{200}$ D $\sqrt{32}$ E $\sqrt{8}$

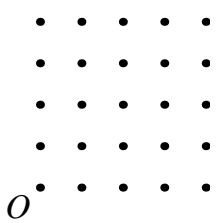
14. What is the smallest number of rectangles, each measuring 2 cm by 3 cm, which are needed to fit together without overlap to form a rectangle whose sides are in the ratio 5 : 4 ?

A 10 B 15 C 20 D 30 E 60

15. Three dice, each showing numbers 1 to 6, are coloured red, blue and yellow respectively. Each of the dice is rolled once. The total of the numbers rolled is 10. In how many different ways can this happen?

A 36 B 30 C 27 D 24 E 21

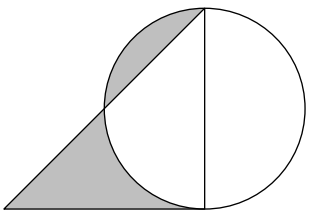
16. An array of 25 equally spaced dots is drawn in a square grid as shown. Point O is in the bottom left corner. Linda wants to draw a straight line through the diagram which passes through O and exactly one other point.



How many such lines can Linda draw?

A 4 B 6 C 8 D 12 E 24

17. A circle of radius r and a right-angled isosceles triangle are drawn such that one of the shorter sides of the triangle is a diameter of the circle.



What is the shaded area?

A $\sqrt{2}r$ B r^2 C $2\pi r$ D $\frac{\pi r^2}{4}$

E $(\sqrt{2}-1)\pi r^2$

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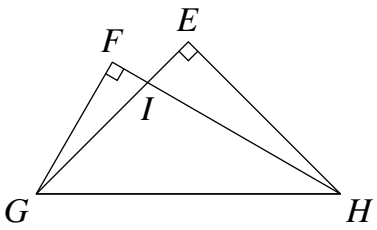
18. The number 840 can be written as $\dfrac{p!}{q!}$, where p and q are positive integers less than 10.

What is the value of $p + q$?

Note that, $n! = 1 \times 2 \times 3 \times \cdots \times (n-1) \times n$.

A 8 B 9 C 10 D 12 E 15

19. The diagram shows two overlapping triangles: triangle FGH with interior angles 60°, 30° and 90° and triangle EGH which is a right-angled isosceles triangle.



What is the ratio of the area of triangle IFG to the area of triangle IEH?

A 1 : 1 B 1 : $\sqrt{2}$ C 1 : $\sqrt{3}$ D 1 : 2 E 1 : 3

20. Laura and Dina have a running race. Laura runs at constant speed and Dina runs n times as fast where $n > 1$. Laura starts s m in front of Dina.

What distance, in metres, does Dina run before she overtakes Laura?

A $\dfrac{ns}{n-1}$ B ns C $\dfrac{s}{n-1}$ D $\dfrac{ns}{n+1}$ E $\dfrac{s}{n}$

21. The numbers m and k satisfy the equations $2^m + 2^k = p$ and $2^m - 2^k = q$.

What is the value of 2^{m+k} in terms of p and q?

A $\dfrac{p^2 - q^2}{4}$ B $\dfrac{pq}{2}$ C $p + q$ D $\dfrac{(p-q)^2}{4}$ E $\dfrac{p+q}{p-q}$

22. A triangle with interior angles 60°, 45° and 75° is inscribed in a circle of radius 2.

What is the area of the triangle?

A $2\sqrt{3}$ B 4 C $6 + \sqrt{3}$ D $6\sqrt{3}$ E $3 + \sqrt{3}$

23. Let x be a real number. What is the minimum value of $\left(x^2 - 4x + 3\right)\left(x^2 + 4x + 3\right)$?

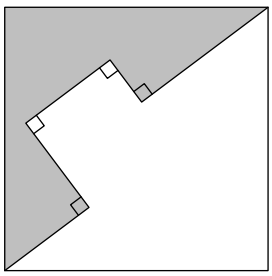
A -16 B -9 C 0 D 9 E 16

24. Saba, Rayan and Derin are cooperating to complete a task. They each work at a constant rate independent of whoever else is working on the task. When all three work together, it takes 5 minutes to complete the task. When Saba is working with Derin, the task takes 7 minutes to complete. When Rayan is working with Derin, the task takes 15 minutes to complete.

How many minutes does it take for Derin to complete the task on his own?

A 21 B 28 C 35 D 48 E 105

25. Five line segments of length 2, 2, 2, 1 and 3 connect two corners of a square as shown in the diagram.



What is the shaded area?

A 8 B 9 C 10 D 11 E 12

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